***Question 11***

***It might be worthwhile here to draw a timeline and identify the corresponding tupling periods.***

**a)** (In class) If Sparky’s height 3-tuples *then* 5-tuples, overall his height will experience a\_\_\_\_\_\_\_-tupling.

Answer: 15

**b)** (In class) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to 3-tuple?

Answer: log2(3)

**c)** (In class) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to 5-tuple?

Answer: log2(5)

**d)** (In class) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to 15-tuple?

Answer: log2(3)+ log2(5); log2(15)

**e)** (In class) Write an equation using logarithmic notation representing the relationship between these three values.

Answer: log2(3)+ log2(5)=log2(15)

**f.** (By teacher) Now, discuss how your equations would change had you measured in: 2-week periods, 1-day periods, 1-year periods, *b*-tupling periods

Answer: The base values would change to:

4,

2^(1/7) or 1.1,

^52,

b

***Question 12***

**a.** (Exercise) If Sparky’s height 34-tuples *then* 57-tuples, overall his height will experience a   
\_\_\_\_\_\_\_-tupling.

**b.** (Exercise) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to 34-tuple?

**c.** (Exercise) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to 57-tuple?

**d.** (Exercise) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to -tuple?

**e.** (Exercise) Write an equation using logarithmic notation representing the relationship between these three values.

**f.** (Exercise) If Sparky’s height *X*-tuples *then* *Y*-tuples, overall his height will experience a   
\_\_\_\_\_\_\_\_-tupling.

**g.** (Exercise) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to *X*-tuple?

**h.** (Exercise) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to *Y*-tuple?

**i.** (Exercise) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to *XY*-tuple?

**j.** (Exercise) Write an equation using logarithmic notation representing the relationship between these three values.

***Logarithmic Property 2***

**LP2:** The number of *b*-tupling periods needed to result in an *X/Y*-tupling is the same as the number of *b*-tupling periods needed to result in an *X*-tupling minus the number of *b*-tupling periods needed to result in a *Y*-tupling.

***Question 13***

**a.** (In class)

**(A)** At some point in time, **(B)** After some time, Sparky’s **(C)** After a total of 3.3219 weeks,

Sparky the cactus was height grew by some factor. Sparky’s height 10-tupled in size

this tall. {2nd:}Draw the resulting Sparky. from point (A).

 From (B) to (C) he 2-tupled over 1 week. {1st:} Draw the resulting Sparky.

**b.** (In class)By what factor did Sparky grow from point (A) to point (B)? How long did it take Sparky to grow by this factor?

***Question 14***

***It might be worthwhile here to draw a timeline and identify the corresponding tupling periods.***

**a)** (In class) Suppose in some unknown amount of time, Sparky’s height \_\_\_\_\_\_\_\_-tuples. 2 weeks (2-tupling periods) later, Sparky’s height 4-tupled in size. If overall, Sparky’s height 12-tupled in size over a 3.585 week (2-tupling) period, how long does it take for Sparky’s height to experience the \_\_\_\_\_\_\_\_-tupling?

Answer: 3; 1.585 weeks

**b)** (In class) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to 12-tuple?

Answer: 3.585, log2(12)

**c)** (In class) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to 4-tuple?

Answer: 2, log2(4)

**d)** (In class) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to -tuple?

Answer: 1.585, log2(3)

Or log2(12) – log(2)4

**e)** (In class) Write an equation using logarithmic notation representing the relationship between these three values.

Answer: log2(12)- log2(4) = log2(3)

**f.** (By teacher) Now, discuss how your equations would change had you measured in: 3-week periods, 4-day periods, 2-year periods, *b*-tupling periods.

Answer: The base values would change to:

8,

2^(4/7),

2^104,

b

***Question 15***

**a.** (Exercise) Suppose in some unknown amount of time, Sparky’s height \_\_\_\_\_\_\_\_-tuples. 1 week (2-tupling periods) later, Sparky’s height 2-tupled in size. If overall, Sparky’s height 34-tupled in size over a 5.087 week (2-tupling) period, how long does it take for Sparky’s height to experience the \_\_\_\_\_\_\_\_-tupling?

**b.** (Exercise) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to 34-tuple?

**c.** (Exercise) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to 2-tuple?

**d.** (Exercise) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to \_\_\_\_\_-tuple?

**e.** (Exercise) Write an equation using logarithmic notation representing the relationship between these three values.

***Question 16***

**a.** (Exercise) If Sparky’s height *Y*-tuples *then* *\_\_\_\_\_\_\_*-tuples, overall his height will experience a *X*-tupling.

**b.** (Exercise) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to *X*-tuple?

**c.** (Exercise) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to *Y*-tuple?

**d.** (Exercise) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to X/Y-tuple?

**e.** (Exercise) Write an equation using logarithmic notation representing the relationship between these three values.

***Logarithmic Property 3***

**LP3:** The number of *b*-tupling periods needed to experience an -tupling is *y* times as large as the number of *b*-tupling periods needed to experience an *X*-tupling.

***Question 17***

**a.** (By teacher) Suppose we observed Sparky’s height 8-tuple in size two times in a row. On the paper provided, document Sparky’s height at these moments. Represent and determine the overall growth factor for this situation.

Answer: It might be worthwhile here to also identify the corresponding tupling periods.

The overall growth factor is **8^2**

**b.** (In class) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to 8-tuple?

Answer: log2(8)

**c.** (In class) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to 8^2-tuple?

Answer: log2(8^2)

Or 2 \* log2(8)

**d.** (In class) Write an equation using logarithmic notation representing the relationship between these two values.

Answer: log2(8^2) = 2log2(8)

***Question 18***

**a.** (In class) If fifty 4-tupling periods elapse, overall Sparky will \_\_\_\_\_\_\_-tuple.

Answer: 4^50

**b.** (In class) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to 4-tuple?

Answer: log2(4)

**c.** (In class) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to \_\_\_\_\_\_\_-tuple?

Answer: log2(4^50) OR 50 \* log2(4)

**d.** (In class) Write an equation using logarithmic notation representing the relationship between these two values.

Answer: log2(4^50)=50log2(4)

***Question 19***

**a.** (Exercise) If y *X*-tupling periods elapse, overall Sparky will \_\_\_\_\_\_\_-tuple.

**b.** (Exercise) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to *X*-tuple?

**c.** (Exercise) How many 2-tupling (1-week) periods need to elapse for Sparky’s height to *\_\_\_\_\_\_\_*-tuple?

**d.** (Exercise) Write an equation using logarithmic notation representing the relationship between these two values.

**e.** (Exercise) Now, discuss how your equations would change had you measured in days, 5-week periods, 2-day periods, ½-year periods, *b*-tupling periods

***Logarithmic Property 4***

**LP4: (**Change of Base) The *x*-tupling period will always be *k* times as large as the *b*-tupling period (this value does not depend on the unit chosen to measure both the *x*- and *b*-tupling periods).

***Question 20***

**a.** (In class) Suppose we observed Sparky’s height 2-tuple in size five times in a row. Determine the overall growth factor for this situation.

Answer: 2^5

*On the paper provided, identify the 2-tupling period and the 2^5-tupling period.*

**b.** (In class) The 2^5-tupling period is how many times as large as the 2-tupling period.

Answer: 5

*What unit are you using to measure the tupling periods in this situation? Will this affect your answer?*

**c.** (In class) How many 4-tupling (2-week) periods need to elapse for Sparky’s height to 2-tuple?

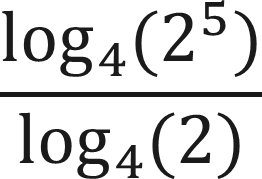
Answer: log4(2)

**d.** (In class) How many 4-tupling (2-week) periods need to elapse for Sparky’s height to 2^5-tuple?

Answer: log4(2^5)

**e.** (In class) Using this information, determine how many times as large the 2^5-tupling period is compared to the 2-tupling period.

Answer: log4(2^5)/ log4(2)



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**f.** (In class) How many 10-tupling periods need to elapse for Sparky’s height to 2-tuple?

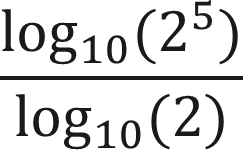
Answer: log10(2)

**g.** (In class) How many 10-tupling periods need to elapse for Sparky’s height to 2^5-tuple?

Answer: log10(2^5)

**h.** (In class) Using this information, determine how many times as large the 2^5-tupling period is compared to the 2-tupling period.

Answer:



**i.** (In class) What conclusions can you make?

Answer: The measuring stick (base value) doesn’t matter

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**j.** (In class) The 15-tupling period is how many times as large as the 10-tupling period?

Answer: log(15)/log(10) *base values may vary – as long as they are the same*

**k.** (In class) Discuss how your answers would change had you measured in: 6-day periods, 13-week periods, 6-tupling periods, 10-tupling periods.

Answer: The base values would change to:

2^(6/7),

2^13,

6,

10

**l.** (In class) In general, the *X*-tupling period is \_\_\_\_\_\_\_\_\_\_\_ times as large as the *Y*-tupling period.

Answer: logb(X)/logb(Y)

**m.** (In class) Develop an equation relating , , and (for b, c, X, Y > 0)

Answer:

**n.** (In class) How does your equation above relate to ?

Answer: Both sides are equal to , but WHY?

Because:

***Logarithmic Property 5***

**LP5:** (In class) If a value b-tuples times (the number of b-tupling periods needed to result in an x-tupling) the value will x-tuple.

***Question 21***

**a.** (In class) Fill in the exponent/box to make the statement true (no calculator): 7.2 = 64.3

Answer:

7.2 ^ (?) = 7.2 ^ *(the number of 7.2-tupling periods to 64.3-tuple)* = 64.3

**b.** (In class) What does the exponent represent in this situation?

Answer: The number of 7.2-tupling periods needed to 64.3-tuple.

**c.** (In class) Evaluate: 

Answer: 17

**d.** (In class) Simplify: 

Answer: x

***Some tests:***

**Sparky the Saguaro:** Emily purchased the mystical cactus shown in the video (Geogebra Applet) on Sunday, January 1st and named the saguaro Sparky. She decided to record the displayed time-lapse video of Sparky’s growth and noticed he was growing in a peculiar way, Sparky’s height 2-tuples in size each week.

**1.** (Exercise) Over some amount of time, Sparky’s height varied from 12 feet to 43 feet. Therefore, in this unknown amount of time, Sparky’s height:

* 1. 12-tupled in size, or became 12 times as large
  2. 12/43-tupled in size, or became 12/43 times as large
  3. 43/12-tupled in size, or became 43/12 times as large
  4. 43-tupled in size
  5. None of the above.

**2.** (Exercise) Over some amount of time, Sparky’s height varied from 12 feet to 43 feet. If we wish to determine the number of weeks it takes for Sparky’s height to reach 43 feet (from 12 feet), we wish to determine ….. , and the number of weeks will be ….. :

* 1. The number of 43/12-tupling periods it takes to 2-tuple;
  2. The number of 2-tupling periods it takes to 43/12-tuple;
  3. The number of 2-tupling periods it takes to 43-tuple;
  4. The number of 43-tupling periods it takes to 2-tuple;
  5. None of the above.

**3.** (Exercise) Over some amount of time, Sparky’s height varied from 12 feet to 43 feet. If we wish to determine the number of *2-week periods* it takes for Sparky’s height to reach 43 feet (from 12 feet), we wish to determine ….. , and the number of *2-week periods* will be ….. :

1. The number of 43/12-tupling periods it takes to 4-tuple;
2. The number of 4-tupling periods it takes to 43/12-tuple;
3. The number of 4-tupling periods it takes to 43-tuple;
4. The number of 43-tupling periods it takes to 4-tuple;
5. None of the above.